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Real-time Systems

Manipulator Control

Outline

1. Control Theory Review
2. Joint-level PD Control
3. Computed Torque Method
4. Non-linear Feedback Control
5. Servo Control

Manipulator Dynamics Revisit

- Dynamics Model of n-link Arm

$$\tau = D(q)\ddot{q} + H(q,\dot{q}) + C(q)$$

The Acceleration-related Inertia term, Symmetric Matrix

The Coriolis and Centrifugal terms

The Gravity terms

$$\tau = \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_n \end{bmatrix}$$

Driving torque applied on each link

Non-linear, highly coupled , second order differential equation

Joint torque ⟺ Robot motion

Jacobian Matrix Revisit

Forward Kinematics

$$T_0^6 = \begin{bmatrix} n & s & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times4}$$

$$p = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} h_1(q) \\ h_2(q) \\ h_3(q) \end{bmatrix}$$

$$\{n, s, a\} \rightarrow \begin{bmatrix} \phi(q) \\ \theta(q) \\ \psi(q) \end{bmatrix} = \begin{bmatrix} h_4(q) \\ h_5(q) \\ h_6(q) \end{bmatrix}$$

$$Y_{6\times1} = h(q) = \begin{bmatrix} h_1(q) \\ h_2(q) \\ \vdots \\ h_6(q) \end{bmatrix}$$

$$\dot{Y}_{6\times1} = J_{6\times n}\dot{q}_{n\times1}$$

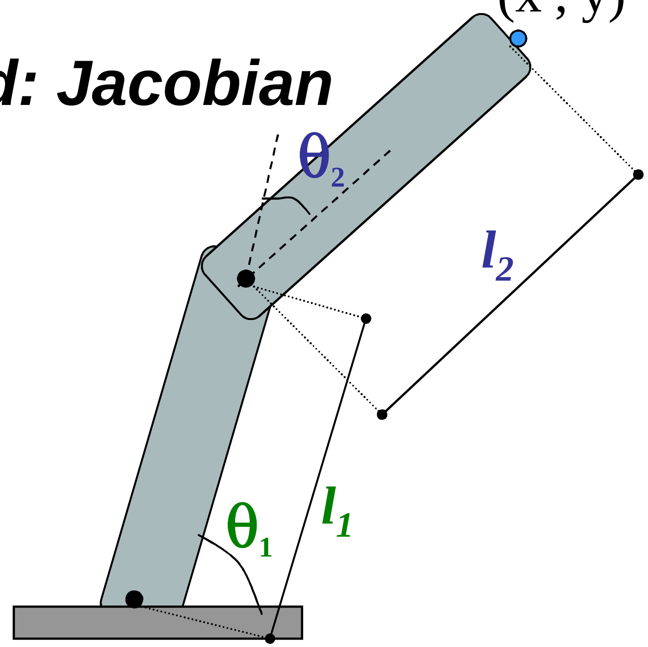
$$J = \left(\frac{dh(q)}{dq}\right)_{6\times n} = \begin{bmatrix} \frac{\partial h_1}{\partial q_1} & \frac{\partial h_1}{\partial q_2} & \cdots & \frac{\partial h_1}{\partial q_n} \\ \frac{\partial h_2}{\partial q_1} & \frac{\partial h_2}{\partial q_2} & \cdots & \frac{\partial h_2}{\partial q_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial h_6}{\partial q_1} & \frac{\partial h_6}{\partial q_2} & \cdots & \frac{\partial h_6}{\partial q_n} \end{bmatrix}_{6\times n}$$

$$\dot{Y} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \left[\frac{dh(q)}{dq}\right]_{6\times n} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}_{n\times1}$$

Jacobian Matrix Revisit

Example: 2-DOF planar robot arm

Given l_1, l_2 , *Find: Jacobian*



$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \end{bmatrix} = \begin{bmatrix} h_1(\theta_1, \theta_2) \\ h_2(\theta_1, \theta_2) \end{bmatrix}$$

$$\dot{Y} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

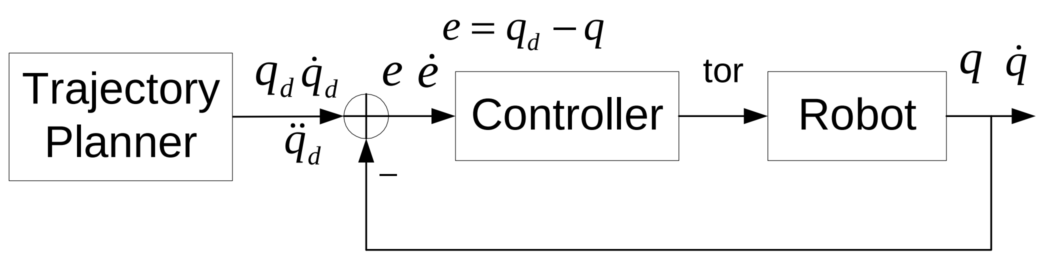
$$J = \begin{bmatrix} \dfrac{\partial h_1}{\partial \theta_1} & \dfrac{\partial h_1}{\partial \theta_2} \\ \dfrac{\partial h_2}{\partial \theta_1} & \dfrac{\partial h_2}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} -l_1 \sin\theta_1 - l_2 \sin(\theta_1 + \theta_2) & -l_2 \sin(\theta_1 + \theta_2) \\ l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) & l_2 \cos(\theta_1 + \theta_2) \end{bmatrix}$$

Robot Manipulator Control

- Robot System:

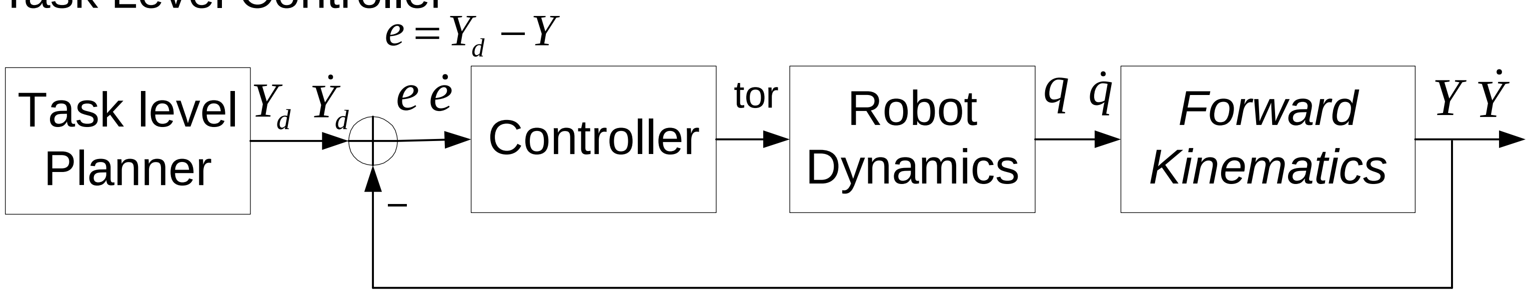
$$\begin{cases} D(q)\ddot{q} + H(q,\dot{q}) + C(q) = \tau \\ Y = h(q) \end{cases}$$

Joint Level Controller



Find a control input (tor), $q \rightarrow q_d \quad as \quad t \rightarrow \infty$

- Task Level Controller



Find a control input (tor), $Y \rightarrow Y_d \quad as \quad t \rightarrow \infty \quad e = Y_d - Y \rightarrow 0$

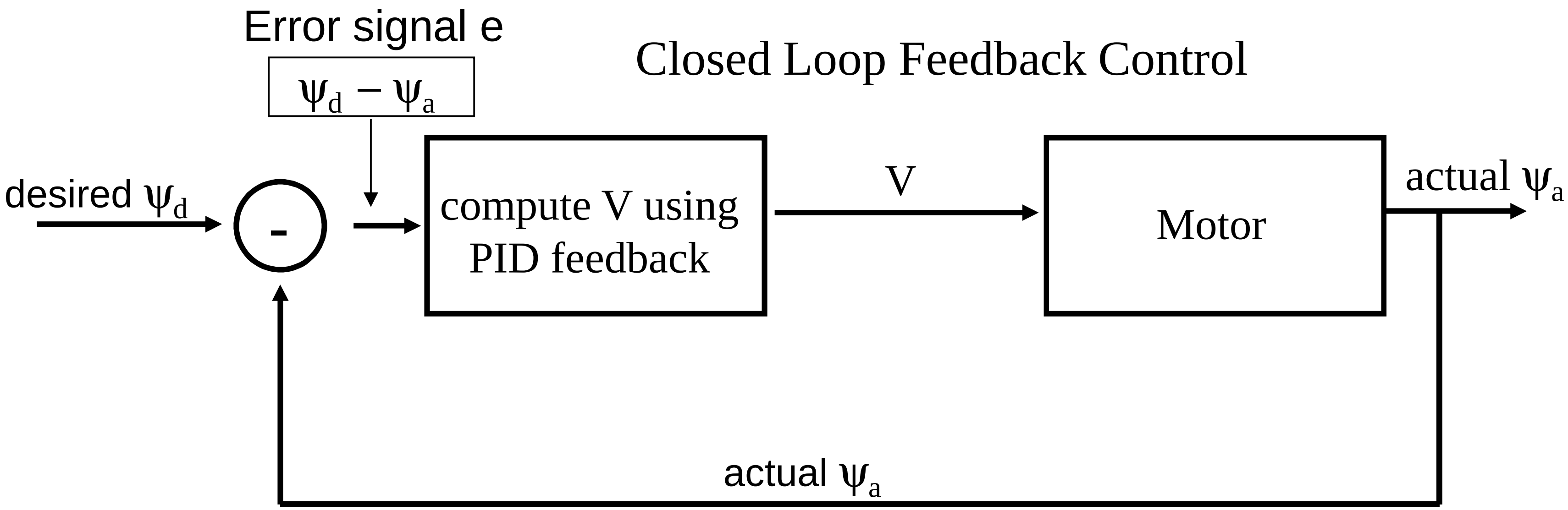
Control Theory Review

PID controller: Proportional / Integral / Derivative control

$$e = \psi_d - \psi_a$$

$$V = K_p \cdot e + K_i \int e \, dt + K_d \frac{de}{dt}$$

Closed Loop Feedback Control



Control Theory Review

Linear Control System

State space equation of a system

$$\dot{x} = Ax + Bu \qquad \text{(Equ. 1)}$$

Example: a system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u \end{cases} \Longrightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Eigenvalue of A are the root of characteristic equation

$$|\lambda I - A| = 0 \qquad |\lambda I - A| = \begin{vmatrix} \lambda & -1 \\ 0 & \lambda \end{vmatrix} = \lambda^2 = 0$$

Asymptotically stable $\Longleftrightarrow$ all eigenvalues of A have negative real part

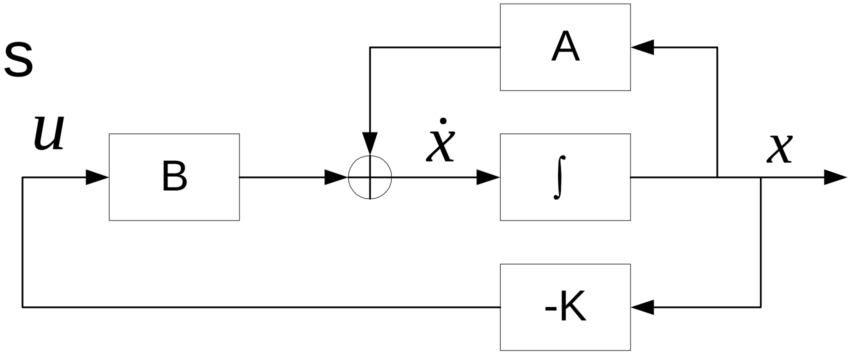
Control Theory Review

Find a state feedback control $u = -K \cdot x$ such that the closed loop system is asymptotically stable

$$u = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \text{(Equ. 2)}$$

Closed loop system becomes

$$\dot{x} = (A - BK)x$$



Chose K, such that all eigenvalues of A'=(A-BK) have negative real parts

$$\left|\lambda I - A'\right| = \begin{vmatrix} \lambda & -1 \\ k_1 & \lambda + k_2 \end{vmatrix} = \lambda^2 + k_2\lambda + k_1 = 0$$

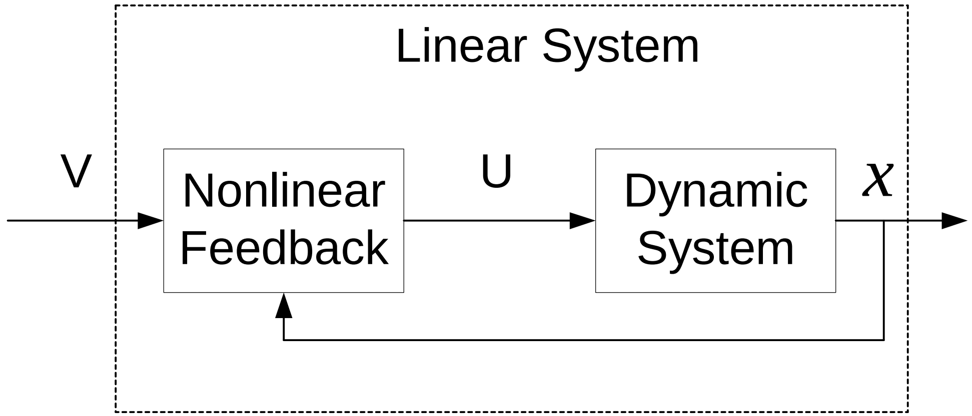
Control Theory Review

Feedback linearization

Nonlinear system $\dot{X} = f(x) + G(x)U$

$$U = [-G^{-1}(x) f(x) + G^{-1}(x) V]$$

$$\dot{X} = V$$



Example:

Original system:

$$\ddot{x} + \cos x = U$$

Nonlinear feedback:

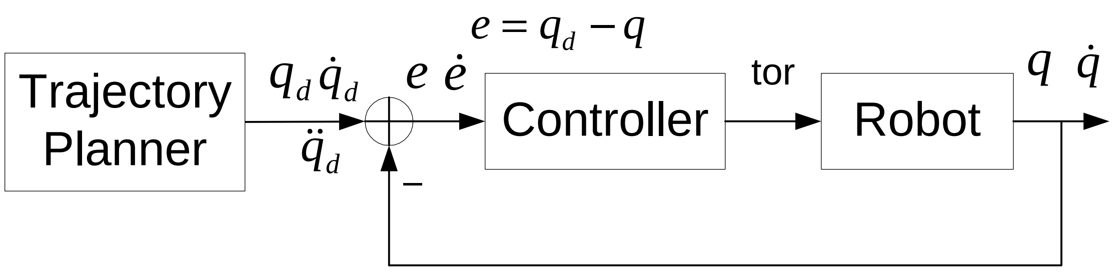
$$U = \cos x + V$$

Linear system: $\ddot{x} = V$

Robot Motion Control

Joint level PID control

- each joint is a servo-mechanism
- adopted widely in industrial robot
- neglect dynamic behavior of whole arm
- degraded control performance especially in high speed
- performance depends on configuration



Robot Motion Control

Computed torque method

Robot system:
$$\begin{cases} D(q)\ddot{q} + H(q,\dot{q}) + C(q) = \tau \\ Y = h(q) \end{cases}$$

Controller:
$$tor = D(q)[\ddot{q}^d + k_v(\dot{q}^d - \dot{q}) + k_p(q^d - q)] + H(q,\dot{q}) + C(q)$$

$$(\ddot{q}^d - \ddot{q}) + k_v(\dot{q}^d - \dot{q}) + k_p(q^d - q) = 0$$

Error dynamics $\ddot{e} + k_v\dot{e} + k_p e = 0$

Advantage: compensated for the dynamic effects

Condition: robot dynamic model is known

Robot Motion Control

Error dynamics $\ddot{e} + k_v \dot{e} + k_p e = 0$

Define states: $x_1 = e$, $x_2 = \dot{e}$ ⇒ $\dot{x}_1 = x_2$, $\dot{x}_2 = -k_v x_2 - k_p x_1$

In matrix form: $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_p & -k_v \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = AX$$

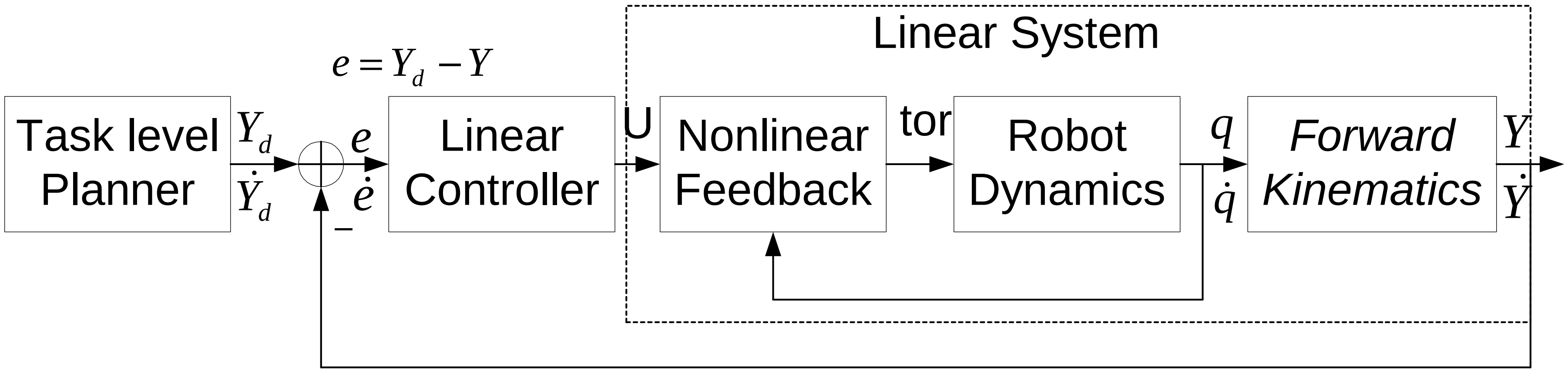
Characteristic equation: $$|\lambda I - A| = \begin{vmatrix} \lambda & -1 \\ k_p & \lambda + k_v \end{vmatrix} = \lambda^2 + k_v \lambda + k_p = 0$$

The eigenvalue of A matrix is: $$\lambda_{1,2} = \frac{-k_v \pm \sqrt{k_v^{\ 2} - 4k_p}}{2}$$

Condition: λ have negative real part ⇒ One of a selections: $k_v > 0$, $k_p > 0$

Robot Motion Control

- Non-linear Feedback Control



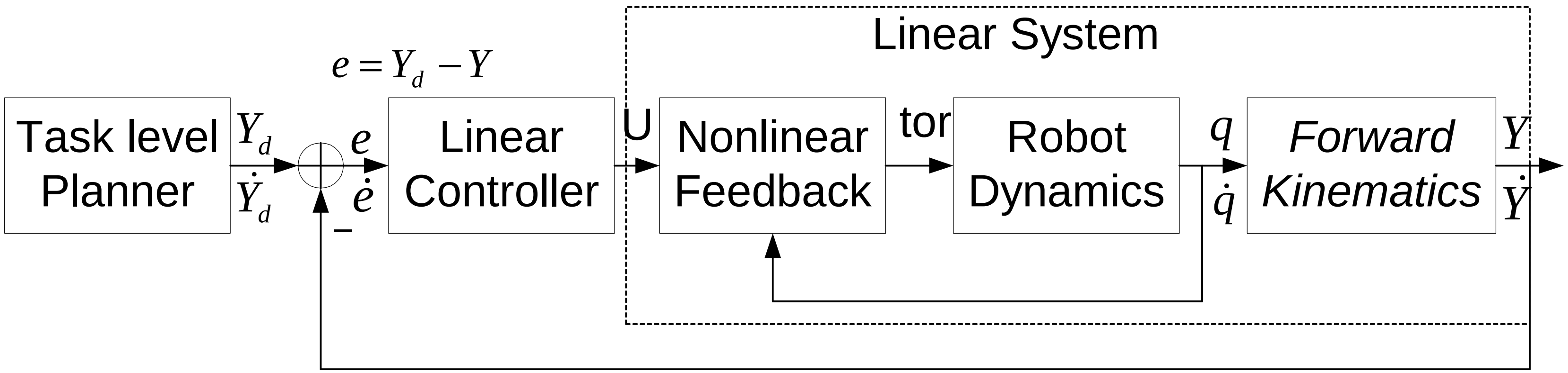
Robot System: $$\begin{cases} D(q)\ddot{q} + H(q,\dot{q}) + C(q) = \tau \\ Y = h(q) \end{cases}$$

Jocobian: $\dot{Y} = \frac{d}{dq}[h(q)]\cdot\dot{q} = J\dot{q}$ ⇨ $\ddot{Y} = \dot{J}\dot{q} + J\ddot{q}$ ⇨ $\ddot{q} = J^{-1}(\ddot{Y} - \dot{J}\dot{q})$

$$D(q)J^{-1}(\ddot{Y} - \dot{J}\dot{q}) + H(q,\dot{q}) + C(q) = \tau$$

Robot Motion Control

- Non-linear Feedback Control



Design the nonlinear feedback controller as:

$$tor = D(q)J^{-1}(U - \dot{J}\dot{q}) + H(q,\dot{q}) + C(q)$$

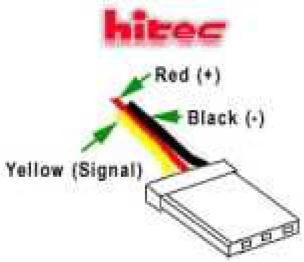
Then the linearized dynamic model:

$$D(q)J^{-1}\ddot{Y} = D(q)J^{-1}U \quad \Rightarrow \quad \ddot{Y} = U$$

Design the linear controller: $U = \ddot{Y}_d + k_v(\dot{Y}_d - \dot{Y}) + k_p(Y_d - Y)$

Error dynamic equation: $\ddot{e} + k_v\dot{e} + k_p e = 0$

Servo Control

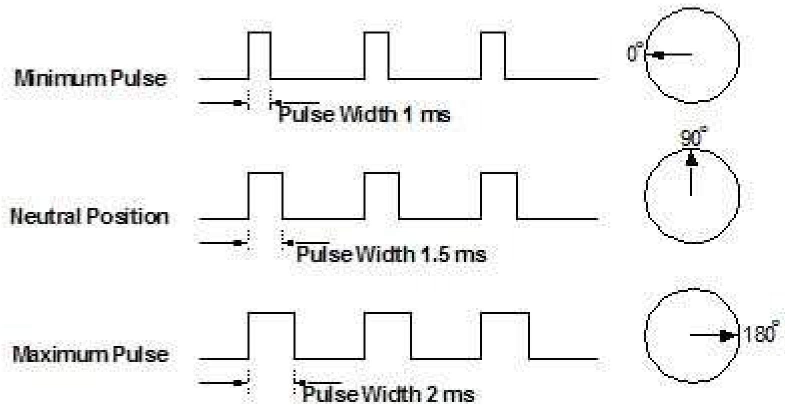


Servo Wiring
All servos have three wires:
Black or **Brown** is for ground.
Red is for power (~4.8-6V).
Yellow, **Orange**, or **White** is the **signal** wire (3-5V).

Servo Control

Servos are DC motors with built in **gearing** and **feedback control loop circuitry.** And no motor drivers required!

The standard **time vs. angle** is represented in this chart:



Servo Control

The servo turn rate, or **transit time**, is used for determining servo rotational velocity. This is the amount of time it takes for the servo to move a set amount, usually 60 degrees. For example, suppose you have a servo with a transit time of 0.17sec/60 degrees at no load. This means it would take nearly half a second to rotate an entire 180 degrees. More if the servo were under a load. This information is very important if high servo response speed is a requirement of your robot application. It is also useful for determining the maximum forward velocity of your robot if your servo is modified for full rotation. Remember, the worst case turning time is when the servo is at the minimum rotation angle and is then commanded to go to maximum rotation angle, all while under load. This can take several seconds on a very high torque servo.